Session 6.1

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Notes to keep in mind

Make sure you have these things in your notes, because I will refer to them with the expectation that you have learned, memorized, or written them down.

1. Solving a system of equations with the **elimination method**

$$\begin{cases} 4x - 7y = -12 \\ -3x + 6y = 9 \end{cases} \xrightarrow{\text{multiply}} \begin{cases} 12x - 21y = -36 \\ -12x + 24y = 36 \end{cases} \xrightarrow{\text{add}} 3y = 0 \xrightarrow{\text{solve}} \boxed{y = 0} \xrightarrow{\text{plug in}} -3x+6(0) = 9 \xrightarrow{\text{solve}} \boxed{x = -3}$$

2. Factoring a polynomial from $x^2 + b * x + c$ into $(x + u)(x + v)$,
   (a) Remember that $b = u + v$ and $c = u * v$
   (b) Start by factoring out c, such as $24 = 1 * 24 = 2 * 12 = 3 * 8 = 4 * 6$
   (c) See if any pair of factors add up to equal b
   (d) If c is positive, that means u and v are both either positive or negative
   (e) If c is negative, one is positive and the other is negative

Main problems

1. Find the (x, y) solution to each of the following:

   (a) $\begin{cases} 7x - 8y = -1 \\ y = 5x - 4 \end{cases}$
   (b) $\begin{cases} -11x - 6y = 9 \\ y = -2x + 3 \end{cases}$
   (c) $\begin{cases} -2x - 3y = -7 \\ y = 6x - 11 \end{cases}$
   (d) $\begin{cases} -4x + 5y = -13 \\ y = -7x + 13 \end{cases}$
   (e) $\begin{cases} 3x + 12y = -15 \\ x = 8y - 2 \end{cases}$
   (f) $\begin{cases} -2x - 10y = -2 \\ x = 5y - 13 \end{cases}$

2. Find the (x, y) solution to each of the following:

   (a) $\begin{cases} 3x + 5y = -35 \\ 6x + 6y = -54 \end{cases}$
   (b) $\begin{cases} 6x + 4y = 6 \\ 2x + 4y = 2 \end{cases}$
   (c) $\begin{cases} 3x + 6y = 21 \\ 4x + 2y = 4 \end{cases}$
   (d) $\begin{cases} 4x + 4y = 4 \\ 6x + 2y = -2 \end{cases}$
   (e) $\begin{cases} 2x + 4y = -14 \\ 5x + 3y = -21 \end{cases}$
   (f) $\begin{cases} 6x + 2y = -6 \\ 4x + 3y = -9 \end{cases}$
   (g) $\begin{cases} 4x + 3y = -7 \\ 3x + 5y = -19 \end{cases}$
   (h) $\begin{cases} 5x + 6y = -37 \\ 3x + 5y = -25 \end{cases}$
   (i) $\begin{cases} 6x + 2y = 10 \\ 4x + 3y = 5 \end{cases}$

3. For each of the following quadratic polynomials, **either** describe all of the transformations, **or** graph it and label five points. If you describe the transformations (how the graph differs from $y = x^2$), use phrases like, *"nothing"*, or *"up 2, then left 4, then reflected about x-axis"*.

(a) $y = x^2$
(b) $y = -x^2$
(c) $y = x^2 + 4$
(d) $y = x^2 - 3$
(e) $y = -x^2 - 2$
(f) $y = (x - 2)^2$
(g) $y = (x + 1)^2$
(h) $y = -(x + 3)^2$
(i) $y = 2x^2$
(j) $y = 1/2 * x^2$
(k) $y = (x + 5)^2 - 9$
(l) $y = (x - 4)^2 + 6$
(m) $y = -(x + 6)^2 + 10$
(n) $y = -(x - 3)^2 - 7$
(o) $y = -3(x - 7)^2$
(p) $y = 1/4 * (x - 1)^2 + 5$
(q) $y = -5(x + 4)^2 - 2$
(r) $y = (3x + 6)^2 + 1$

4. For each of the following transformations to $y = x^2$, write the quadratic equation in the form $y = c * (x + a)^2 + b$.

(a) Up 4
(b) Down 2
(c) Left 1
(d) Right 5
(e) Reflect about x-axis
(f) Up 2, then right 3
(g) Down 4, then left 5
(h) Reflect about x-axis, then right 1
(i) Reflect about x-axis, then up 4, then left 2
(j) Up 4, then reflect about x-axis
(k) Down 7, then reflect about x-axis, then right 3
(l) Up 4, then reflect about x-axis

5. Expand each of the following polynomials:

(a) $(x + 2)^2$
(b) $(x - 7)^2$
(c) $(x - 5)^2$
(d) $(x + 9)^2$
(e) $(x - 12)^2$
(f) $(x + 11)^2$
(g) $2(x + 3)^2$
(h) $3(x - 1)^2$

6. Factor each of the following:

(a) $y = x^2 + 6x + 9$
(b) $y = x^2 - 14x + 49$
(c) $y = x^2 - 18x + 81$
(d) $y = x^2 + 12x + 36$
(e) $y = x^2 + 24x + 144$
(f) $y = x^2 - 22x + 121$
(g) $y = 3x^2 - 30x + 75$
(h) $y = -4x^2 + 24x - 36$

7. Complete the squares of each graph, and describe the transformations happening in words:

(a) $x^2 - 6x + 14$
(b) $x^2 + 4x + 11$
(c) $x^2 + 2x + 10$
(d) $x^2 - 14 + 40$
(e) $x^2 - 12x + 12$
(f) $x^2 + 2x - 4$
(g) $x^2 - 6x - 6$
(h) $x^2 + 16x - 10$
(i) $x^2 + 24x + 100$
(j) $x^2 + 14x - 9$
(k) $x^2 - 18x + 53$
(l) $x^2 + 8x + 27$
(m) $x^2 + 22x - 21$
(n) $x^2 - 3x + 1$
(o) $-x^2 - 14x + 14$
(p) $-x^2 - 6x + 13$
(q) $4x^2 - 4x + 20$
(r) $2x^2 - 2x + 3$
(s) $-2x^2 + 28x - 7$
(t) $-2x^2 - 2x + 4$
(u) $-3x^2 - 24x + 24$

Counting and probability problems

1. Find the **probability** of drawing each type of card from a standard 52-card poker deck.
   (a) Draw an ace?
   (b) Draw a heart?
   (c) Draw a face card?
2. Suppose you're rolling two dice. How many ways can each event happen?
   (a) Rolling two 6's?
   (b) Rolling a 5 and a 4?
   (c) Rolling two evens?
   (d) Rolling a sum of 3?
   (e) Rolling a sum of 5?
   (f) What is the highest probability sum?
3. How many ways are there to sort each of the following in order?
   (a) Three students
   (b) Four different mugs
   (c) Ten college applications
4. Consider a class of eight students. How many ways can I order them in line with the following restrictions:
   (a) No restrictions?
   (b) Ederson must be in the front of the line?
   (c) Chris must be in the back so I can see where the line ends easily?
   (d) I have Mykal and Jordan stand with each other in line because I find it amusing?
   (e) Ederson, Max, and Enzo insist on standing with each other?
   (f) I need Christian and George to be separated?
5. Suppose you draw two cards in order from a 52-card deck. What is the probability you draw each of the following?
   (a) A 2 and a 7?
   (b) Pair of Ace's?
   (c) Pair of 10's
   (d) Two hearts?
   (e) Two spades in order?
   (f) Any two numbers in order?
   (g) Two cards of different suits?
   (h) Two cards of different numbers?
   (i) Any two numbers not in order?