

## Session 5.2

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### Notes to keep in mind

Make sure you have these things in your notes, because I will refer to them with the expectation that you have learned, memorized, or written them down.

1. Solving a system of equations with the **substitution method**

$$\begin{cases} 5x - 2y = 8 \\ y = x - 1 \end{cases} \xrightarrow{\text{substitute}} 5x - 2(x - 1) = 8 \xrightarrow{\text{solve}} [x = 2] \xrightarrow{\text{plug in}} y = (2) - 1 \xrightarrow{\text{solve}} [y = 1]$$

2. Solving a system of equations with the **elimination method**

$$\begin{cases} 4x - 7y = -12 \\ -3x + 6y = 9 \end{cases} \xrightarrow{\text{multiply}} \begin{cases} 12x - 21y = -36 \\ -12x + 24y = 36 \end{cases} \xrightarrow{\text{add}} 3y = 0 \xrightarrow{\text{solve}} [y = 0] \xrightarrow{\text{plug in}} -3x + 6(0) = 9 \xrightarrow{\text{solve}} [x = -3]$$

3. Factoring a polynomial from  $x^2 + b * x + c$  into  $(x + u)(x + v)$ ,

- (a) Remember that  $b = u + v$  and  $c = u * v$
- (b) Start by factoring out  $c$ , such as  $24 = 1 * 24 = 2 * 12 = 3 * 8 = 4 * 6$
- (c) See if any pair of factors add up to equal  $b$
- (d) If  $c$  is positive, that means  $u$  and  $v$  are both either positive or negative
- (e) If  $c$  is negative, one is positive and the other is negative

### Main problems

1. Solve the following system of equations for the  $(x, y)$  solution

(a)  $\begin{cases} 9x - 4y = 15 \\ y = 3x - 3 \end{cases}$

(e)  $\begin{cases} -2x + 3y = -1 \\ 2x + 5y = 25 \end{cases}$

(i)  $\begin{cases} 2x - y = 9 \\ 3x + 4y = -14 \end{cases}$

(b)  $\begin{cases} -3x + 2y = 15 \\ y = -x + 4 \end{cases}$

(f)  $\begin{cases} 2x + y = 12 \\ -3x + y = 2 \end{cases}$

(j)  $\begin{cases} 4x - 3y = 25 \\ -3x + 8y = 10 \end{cases}$

(c)  $\begin{cases} 7y - 5x = -10 \\ x = -\frac{7}{5}y + 2 \end{cases}$

(g)  $\begin{cases} 2x + y = 9 \\ 3x - y = 16 \end{cases}$

(k)  $\begin{cases} 3x + 4y = 52 \\ 5x + y = 30 \end{cases}$

(d)  $\begin{cases} 5y - 7x = 4 \\ x = \frac{6}{7}y + 5 \end{cases}$

(h)  $\begin{cases} x - 2y = 9 \\ x + 3y = 16 \end{cases}$

(l)  $\begin{cases} 12x - 3y = 6 \\ 4x - y = 2 \end{cases}$

2. Simplify each of the following polynomials

- (a) Add  $-11x^2 - 2x - 15$  to  $3x - 5$       (d) Multiply/expand  $(x - 6)^2$   
 (b) Subtract  $-10x^2 - 10x + 1$  from  $-4x^2 - 15x + 7$       (e) Multiply/expand  $(x - 2)(x - 4)$   
 (c) Subtract  $-14x^2 + 6$  from  $-x^2 - 4x + 9$       (f) Multiply/expand  $(x - 5)(x + 6)$
3. Graph each of the following quadratic polynomials. Describe how the graph differs from  $y = x^2$  using phrases like, “*nothing*”, or “*up 2, then left 4, then reflected about x-axis*”.
- |                     |                          |                            |
|---------------------|--------------------------|----------------------------|
| (a) $y = x^2$       | (f) $y = (x + 2)^2$      | (k) $y = 3x^2$             |
| (b) $y = x^2 + 2$   | (g) $y = -(x + 3)^2$     | (l) $y = 1/2 * x^2$        |
| (c) $y = x^2 - 6$   | (h) $y = 2(x + 5)^2$     | (m) $y = 2(x + 5)^2$       |
| (d) $y = -x^2$      | (i) $y = -(x - 5)^2 - 7$ | (n) $y = 2(x + 5)^2$       |
| (e) $y = (x - 4)^2$ | (j) $y = (x + 3)^2 + 5$  | (o) $y = -(4x + 12)^2 - 3$ |
4. For each of the following transformations to  $y = x^2$ , write the quadratic equation in some form.
- |                            |  |
|----------------------------|--|
| (a) Up 3                   | (i) Left 3, then reflected about x-axis                              |
| (b) Down 7                 | (j) Down 4, then reflected about x-axis                              |
| (c) Right 2                | (k) Left 13, then up 7, then reflected about x-axis                  |
| (d) Left 5                 | (l) Up 4, then left 13, then vertical stretch by 2                   |
| (e) Left 3, then down 7    | (m) Reflected about x-axis, then right 4, vertical compress by 3     |
| (f) Right 3, then up 4     | (n) Down 6, then vertical compress by 2, then reflected about x-axis |
| (g) Left 2, then down 5    |  |
| (h) Reflected about x-axis |  |
5. Complete the squares of each graph, and describe the shift happening in words.
- |                     |                      |                        |
|---------------------|----------------------|------------------------|
| (a) $x^2 + 4x + 20$ | (g) $x^2 - 14x + 20$ | (m) $4x^2 - 24x + 20$  |
| (b) $x^2 + 6x + 12$ | (h) $x^2 - 8x - 5$   | (n) $2x^2 - 8x + 3$    |
| (c) $x^2 - 10 + 30$ | (i) $x^2 + 16x + 30$ | (o) $-2x^2 + 10x - 7$  |
| (d) $x^2 - 2x - 15$ | (j) $-x^2 + 4x + 3$  | (p) $x^2 - 3x + 1$     |
| (e) $x^2 + 6x - 5$  | (k) $-x^2 - 8x + 24$ | (q) $-2x^2 - 2x + 4$   |
| (f) $x^2 - 10x + 2$ | (l) $-x^2 - 6x + 7$  | (r) $-3x^2 - 24x + 24$ |
6. Factor each of the following, and list the  $x$ -intercepts:
- |                            |                          |                             |
|----------------------------|--------------------------|-----------------------------|
| (a) $y = x^2 + 6x + 9$     | (j) $y = x^2 - 16$       | (s) $y = x^2 - 2x - 8$      |
| (b) $y = x^2 + 24x + 144$  | (k) $y = 3x^2 - 75$      | (t) $y = x^2 - 14x + 45$    |
| (c) $y = x^2 - 18x + 81$   | (l) $y = 4x^2 - 9$       | (u) $y = x^2 - 18x + 17$    |
| (d) $y = x^2 - 10x + 25$   | (m) $y = 16x^2 - 36$     | (v) $y = x^2 - 3x - 28$     |
| (e) $y = x^2 - 22x + 121$  | (n) $y = x^2 - 144/9$    | (w) $y = x^2 - 8x - 65$     |
| (f) $y = 3x^2 - 12x + 12$  | (o) $y = x^2 - 81/16$    | (x) $y = 3x^2 + 9x - 30$    |
| (g) $y = -2x^2 - 28x - 98$ | (p) $y = x^2 + 10x + 21$ | (y) $y = -2x^2 + 36x - 34$  |
| (h) $y = x^2 - 49$         | (q) $y = x^2 + 13x + 40$ | (z) $y = -4x^2 + 12x + 216$ |
| (i) $y = x^2 - 121$        | (r) $y = x^2 + 14x + 48$ |                             |