

Session 4.3

Mr. Hernandez: josehdz@cs.stanford.edu

Notes to keep in mind

Make sure you have these things in your notes, because I will refer to them with the expectation that you have learned, memorized, or written them down.

1. Solving a system of equations with the **substitution method**

$$\begin{cases} 5x - 2y = 8 \\ y = x - 1 \end{cases} \xrightarrow{\text{substitute}} 5x - 2(x - 1) = 8 \xrightarrow{\text{solve}} \boxed{x = 2} \xrightarrow{\text{plug in}} y = (2) - 1 \xrightarrow{\text{solve}} \boxed{y = 1}$$

2. Solving a system of equations with the **elimination method**

$$\begin{cases} 4x - 7y = -12 \\ -3x + 6y = 9 \end{cases} \xrightarrow{\text{multiply}} \begin{cases} 12x - 21y = -36 \\ -12x + 24y = 36 \end{cases} \xrightarrow{\text{add}} 3y = 0 \xrightarrow{\text{solve}} \boxed{y = 0} \xrightarrow{\text{plug in}} -3x + 6(0) = 9 \xrightarrow{\text{solve}} \boxed{x = -3}$$

3. Solving for x in an absolute value equation, such as $-2|x + 3| + 8 = -10$

- (a) Isolate the absolute value: $-2 * |x + 3| + 8 = -10 \implies -2 * |x + 3| = -18 \implies |x + 3| = 9$
 (b) Identify the two possibilities: $|x + 3| = |9| = |-9| = 9$
 (c) Set the “inside” equal to the “inside”: $x + 3 = 9$ **and** $x + 3 = -9$
 (d) Solve for x : $\boxed{x = 6}$ **and** $\boxed{x = -12}$

4. Factoring a polynomial from $x^2 + b * x + c$ into $(x + u)(x + v)$,

- (a) Remember that $b = u + v$ and $c = u * v$
 (b) Start by factoring out c , such as $24 = 1 * 24 = 2 * 12 = 3 * 8 = 4 * 6$
 (c) See if any pair of factors add up to equal b
 (d) If c is positive, that means u and v are both either positive or negative
 (e) If c is negative, one is positive and the other is negative

Main problems

1. Find all points on the following graphs with the specified value of y

(a) $y = \left| \frac{3}{4}x + \frac{3}{8} \right|$ where $y = 3$
 (b) $y = \left| 3 - \frac{2}{3}x \right|$ where $y = 5$

(c) $y = -\frac{1}{2} * \left| \frac{2}{5}x + 4 \right| - 5$ where $y = -9$

2. For each of the systems of equations, find the (x, y) solution.

(a) $\begin{cases} 13x + 20y = 4 \\ y = -x \end{cases}$

(b) $\begin{cases} -3x - 14y = 1 \\ y = x + 1 \end{cases}$

(c) $\begin{cases} -13x + 4y = -5 \\ y = -x + 3 \end{cases}$

$$\begin{array}{lll}
 \text{(d)} \begin{cases} -4x + 9y = 0 \\ y = 4x - 1 \end{cases} & \text{(g)} \begin{cases} -8y - 10x = 19 \\ x = -\frac{8}{5}y - 2 \end{cases} & \text{(j)} \begin{cases} 7x - y = -11 \\ 2x - 3y = 10 \end{cases} \\
 \text{(e)} \begin{cases} -2x - 5y = 14 \\ y = -3x + 8 \end{cases} & \text{(h)} \begin{cases} -2x - 4y = -2 \\ -4x + y = 14 \end{cases} & \text{(k)} \begin{cases} 3x - 8y = 12 \\ -9x - 2y = 17 \end{cases} \\
 \text{(f)} \begin{cases} 5y - 7x = 4 \\ x = \frac{6}{7}y + 5 \end{cases} & \text{(i)} \begin{cases} -x + 4y = -6 \\ -2x + 5y = -11 \end{cases} & \text{(l)} \begin{cases} 12x - 9y = -3 \\ 3y - 4x - 9 = 0 \end{cases}
 \end{array}$$

3. Simplify each of the following polynomials

$$\begin{array}{ll}
 \text{(a)} \text{ Add } -11x^2 - 2x - 15 \text{ to } 3x - 5 & \text{(d)} \text{ Multiply/expand } (x - 6)^2 \\
 \text{(b)} \text{ Subtract } -10x^2 - 10x + 1 \text{ from } -4x^2 - 15x + 7 & \text{(e)} \text{ Multiply/expand } (x - 2)(x - 4) \\
 \text{(c)} \text{ Subtract } -14x^2 + 6 \text{ from } -x^2 - 4x + 9 & \text{(f)} \text{ Multiply/expand } (x - 5)(x + 6)
 \end{array}$$

4. Graph each of the following quadratic polynomials. Denote the min/max point and x -intercept(s).

$$\begin{array}{lll}
 \text{(a)} y = x^2 & \text{(e)} y = -x^2 & \text{(i)} y = -(x + 3)^2 \\
 \text{(b)} y = x^2 + 2 & \text{(f)} y = 1/2 * x^2 & \text{(j)} y = 2(x + 5)^2 \\
 \text{(c)} y = x^2 - 6 & \text{(g)} y = (x - 4)^2 & \text{(k)} y = -(x - 5)^2 - 7 \\
 \text{(d)} y = 3x^2 & \text{(h)} y = (x + 2)^2 & \text{(l)} y = (x + 3)^2 + 5
 \end{array}$$

5. In general, what happens if we add c (a constant)?

6. In general, what happens if we multiply the polynomial by -1 ?

7. In general, what happens if we add c (a constant) inside the quadratic?

8. Factor each of the following, and list the x -intercepts:

$$\begin{array}{lll}
 \text{(a)} y = x^2 + 6x + 9 & \text{(j)} y = x^2 - 16 & \text{(s)} y = x^2 - 14x + 45 \\
 \text{(b)} y = x^2 + 24x + 144 & \text{(k)} y = 3x^2 - 75 & \text{(t)} y = x^2 - 18x + 17 \\
 \text{(c)} y = x^2 - 18x + 81 & \text{(l)} y = 4x^2 - 9 & \text{(u)} y = x^2 - 3x - 28 \\
 \text{(d)} y = x^2 - 10x + 25 & \text{(m)} y = 16x^2 - 36 & \text{(v)} y = x^2 - 8x - 65 \\
 \text{(e)} y = x^2 - 22x + 121 & \text{(n)} y = x^2 - 144/9 & \text{(w)} y = 3x^2 + 9x - 30 \\
 \text{(f)} y = 3x^2 - 12x + 12 & \text{(o)} y = x^2 - 81/16 & \text{(x)} y = -2x^2 + 36x - 34 \\
 \text{(g)} y = -2x^2 - 28x - 98 & \text{(p)} y = x^2 + 13x + 40 & \text{(y)} y = -4x^2 + 12x + 216 \\
 \text{(h)} y = x^2 - 49 & \text{(q)} y = x^2 + 14x + 48 & \\
 \text{(i)} y = x^2 - 121 & \text{(r)} y = x^2 - 2x - 8 &
 \end{array}$$

More problems

1. Work on 2014 ICTM 1A/2A: <http://www.ilmathcontest.com/hs/Questions/Reg/R14A.pdf>

2. Use the "Noah sheets": <http://teachers.edenpr.org/mkingsbury/mathteam/NoahSheets.pdf>