# Session 3.3

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## Notes to keep in mind

Make sure you have these things in your notes, because I will refer to them with the expectation that you have learned, memorized, or written them down.

1. Two lines are **parallel** if they have the same slope
2. Points are **collinear** if they lie on the same line. *Note:* it is sufficient to check slopes between all the points are equal (think about it!).
3. Two lines are **perpendicular** if the slope of one is the negative inverse of the other.
   (a) Slopes are $m_1$ and $m_2$ and $m_1 = -\dfrac{1}{m_2}$
   (b) The $y$-intercepts don't matter — draw it out and see why it makes sense!
4. Characteristics of a polynomial, such as $ax^2 + bx + c$, or, more generally $ax^n + bx^{n-1} + \ldots + z$
   (a) The **degree** of a polynomial is the highest variable exponent, such as 2 or $n$
   (b) The **leading coefficient** is the coefficient of the variable with the highest degree, such as $a$
   (c) The **constant term** is the number without a variable next to it, such as $c$ or $z$

## Main problems

1. Find all points on the following graphs with the specified value of $y$
   (a) $y = |x + 4|$ where $y = 12$
   (b) $y = |x - 5|$ where $y = 11$
   (c) $y = -|x - 2| + 5$ where $y = 1$
   (d) $y = 2/3 * |x + 3| + 2$ where $y = 12$
2. Find the equation in slope-intercept form for each line, then a parallel one, and a perpendicular one:
   (a) Slope 2 and passes through $(-2, 5)$.
   (b) Slope $-4/3$ and passes through $(6, -6)$.
3. For each set of three points say whether or not they're on the same line, and prove it.
   (a) $(4, 9), (-1, 11)$, and $(-11, 16)$
   (b) $(-2, 7), (-6, 18)$, and $(2, -4)$
4. For each of the systems of equations, find the $(x, y)$ solution.
   (a) $\begin{cases} 3x - 2y = 7 \\ y = 10 \end{cases}$
   (b) $\begin{cases} 7x + 5y = -4 \\ y = -5 \end{cases}$
   (c) $\begin{cases} -2x - 9y = 14 \\ x = 11 \end{cases}$
   (d) $\begin{cases} \frac{2}{3}x - \frac{5}{3}y = -\frac{11}{3} \\ x = -7 \end{cases}$
   (e) $\begin{cases} -3x + 7y = 28 \\ y = x \end{cases}$
   (f) $\begin{cases} 5x - 2y = 8 \\ y = x - 1 \end{cases}$

(g) $\begin{cases} -12y + 5x = 25 \\ x = 2y + 7 \end{cases}$

(h) $\begin{cases} 6x - 8y = 16 \\ 18x - 5y = 10 \end{cases}$

(i) $\begin{cases} 4x - 7y = -12 \\ -3x + 6y = 9 \end{cases}$

(j) $\begin{cases} 2x + 6y = 3 \\ -8x - 4y = 13 \end{cases}$

(k) $\begin{cases} x + 2y = 13 \\ -4x - 2y + 12 = 0 \end{cases}$

(l) $\begin{cases} 3x + 4y = 5 \\ -20y - 15x = -25 \end{cases}$

5. Simplify each of the following polynomials
   (a) Add $6x^2 - 2x - 1$ to $-4x^2 + 7x + 5$
   (b) Subtract $-7x^2 + 3x - 9$ from $5x^2 - 6x - 4$
   (c) Subtract $-7x^2 + 3x - 6$ from $3x^2 + 4x + 4$
   (d) Add $-8x^2 + 11x - 6$ to $-7x^2 - 9x + 14$
   (e) Multiply/expand $(x + 4)(x - 3)$
   (f) Multiply/expand $(x + 5)(x + 7)$
6. Graph each of the following quadratic polynomials. Denote the min/max point and $x$-intercept(s).
   (a) $y = x^2$
   (b) $y = x^2 + 2$
   (c) $y = x^2 - 6$
   (d) $y = 3x^2$
   (e) $y = -x^2$
   (f) $y = \frac{1}{2}x^2$
   (g) $y = (x - 4)^2$
   (h) $y = -(x + 3)^2$
   (i) $y = 2(x + 5)^2$
   (j) $y = -(x - 5)^2 - 7$
   (k) $y = (x + 3)^2 + 5$
7. In general, what happens if we add $c$ (a constant)?
8. In general, what happens if we multiply the polynomial by $-1$?
9. In general, what happens if we add $c$ (a constant) inside the quadratic?
10. Factor each of the following, and list the $x$-intercepts:
    (a) $y = x^2 - 4x + 4$
    (b) $y = x^2 + 10x + 25$
    (c) $y = 2x^2 + 24x + 72$
    (d) $y = x^2 + 3x - 28$
    (e) $y = x^2 + 15x + 36$
    (f) $y = 7x^2 + 7x - 42$
    (g) $y = x^2 + 11x - 12$
    (h) $y = x^2 + 34x + 33$
    (i) $y = 2x^2 - 4x + 70$
    (j) $y = 3x^2 + 30x + 48$
    (k) $y = x^2 - 36$
    (l) $y = x^2 - 16$

## More problems

1. Work on 2014 ICTM AA: http://www.ilmathcontest.com/hs/Questions/Reg/R14AA.pdf
2. Use the "Noah sheets": http://teachers.edenpr.org/ mkingsbury/mathteam/NoahSheets.pdf