Session 3.2

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Notes to keep in mind

Make sure you have these things in your notes, because I will refer to them with the expectation that you have learned, memorized, or written them down.

1. Two lines are **parallel** if they have the same slope
2. Points are **collinear** if they lie on the same line. *Note:* it is sufficient to check slopes between all the points are equal (think about it!).
3. Two lines are **perpendicular** if the slope of one is the negative inverse of the other.
   (a) Slopes are m_1 and m_2 and $m_1 = -\dfrac{1}{m_2}$
   (b) The y-intercepts don't matter — draw it out and see why it makes sense!
4. Characteristics of a polynomial, such as $ax^2 + bx + c$, or, more generally $ax^n + bx^{n-1} + \ldots + z$
   (a) The **degree** of a polynomial is the highest variable exponent, such as 2 or n
   (b) The **leading coefficient** is the coefficient of the variable with the highest degree, such as a
   (c) The **constant term** is the number without a variable next to it, such as c or z

Main problems

1. Find all points on the following graphs with the specified value of y
   (a) $y = |x|$ where $y = 4$
   (b) $y = |x - 3|$ where $y = 8$
   (c) $y = -|x + 3|$ where $y = -11$
   (d) $y = |x - 4| + 9$ where $y = 15$
2. Find the y-intercept and equation of a line that has:
   (a) Slope $3/2$ and passes through $(-2, 6)$.
   (b) Slope $-5/2$ and passes through $(4, -8)$.
3. For each line, write two line equations of one that is parallel, and one that is perpendicular
   (a) $y = \dfrac{4}{7}x - 12$
   (b) $y = -5x + 7$
   (c) $y = \dfrac{13}{9}x + 2$
   (d) $y = 2.\overline{3}x - 3$
4. For each set of three points say whether or not they're on the same line, and prove it.
   (a) $(4, 9), (-1, 11)$, and $(-11, 16)$
   (b) $(-2, 7), (-6, 18)$, and $(2, -4)$
5. For each of the systems of equations, find the (x, y) solution with the substitution method

(a) $\begin{cases} -5x + 2y = 9 \\ y = 7x \end{cases}$ (b) $\begin{cases} 15x + 31y = -3 \\ x = -y + 3 \end{cases}$ (c) $\begin{cases} 10x - 9y = 24 \\ y = x - 2 \end{cases}$

6. For each of the systems of equations, find the (x, y) solution with the elimination method

(a) $\begin{cases} 3x - 4y = 8 \\ 18x - 5y = 10 \end{cases}$ (b) $\begin{cases} 6x - 5y = -32 \\ -7x + 8y = 46 \end{cases}$ (c) $\begin{cases} -2x - 7y = 30 \\ 7x + 4y = 18 \end{cases}$

7. For each of the systems of equations, find the (x, y) solution. If there are no or infinitely many solutions, say that too!

(a) $\begin{cases} 6x - 3y = 5 \\ y - 2x = 8 \end{cases}$ (c) $\begin{cases} 2x = 3y - 1 \\ 6x - 9y = -3 \end{cases}$ (e) $\begin{cases} 5x - 6y = -25 \\ 4x - 3y + 20 = 0 \end{cases}$

(b) $\begin{cases} -4x + 7y + 5 = 0 \\ x - 3y = -5 \end{cases}$ (d) $\begin{cases} y = 3x + 8 \\ 12x - 4y = -32 \end{cases}$

8. Simplify each of the following polynomials
   (a) Add $6x^2 - 2x - 1$ to $-4x^2 + 7x + 5$
   (b) Subtract $-7x^2 + 3x - 9$ from $5x^2 - 6x - 4$
   (c) Subtract $-7x^2 + 3x - 6$ from $3x^2 + 4x + 4$
   (d) Add $-8x^2 + 11x - 6$ to $-7x^2 - 9x + 14$
   (e) Multiply/expand $(x + 4)(x - 3)$
   (f) Multiply/expand $(x + 5)(x + 7)$

9. Graph each of the following quadratic polynomials. Denote the min/max point and x-intercept(s).

(a) $y = x^2$ (e) $y = -x^2$ (h) $y = -(x + 4)^2$
(b) $y = x^2 + 3$ (f) $y = \frac{1}{2}x^2$ (i) $y = 2(x + 4)^2$
(c) $y = x^2 - 4$ (j) $y = (x + 2)^2 - 5$
(d) $y = 2x^2$ (g) $y = (x - 3)^2$ (k) $y = -(x - 5)^2 + 7$

10. In general, what happens if we add 2 to the constant term? subtract 2? add c (a constant)?
11. In general, what happens if we multiply the polynomial by -1?
12. In general, what happens if we add 2 to x in an equation (inside the quadratic)? subtract 3? add c (a constant)?
13. Factor each of the following:

(a) $y = x^2 - 12x + 36$ (d) $y = x^2 - 5x + 6$ (g) $y = 2x^2 - 16x + 30$
(b) $y = x^2 - 22x + 121$ (e) $y = x^2 - 7x + 12$ (h) $y = x^2 - 5x - 14$
(c) $y = x^2 + 8x + 7$ (f) $y = x^2 + 14x + 48$ (i) $y = x^2 - 2x - 48$

More problems

1. Work on: 2013 AMC 10A
2. Use the "Noah sheets" (http://teachers.edenpr.org/ mkingsbury/mathteam/NoahSheets.pdf)