

Session 1.1: Where are you in math?

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By the end of the summer you'll master the material well enough to answer these questions confidently. Just take a breath, relax, and do your thing – I don't expect everyone to get everything right. I'm excited to meet you tomorrow!

Please write your answers on a separate sheet of paper and turn that in. Partial answers are useful for me – you don't need to be 100% right. Please include work, even if it is ugly scratch work. No calculators are allowed. Skip around because problems vary in difficulty.

1 Ordering of real numbers

Please order the following numbers: $\frac{5}{6}$, $\frac{5}{13}$, 0.6, $\frac{1}{6}$, 0.72, $\frac{3}{20}$, $\frac{1.65}{3}$, $\frac{7}{4}$

For each ordered pair, defend your claim with a picture, reformulation, or something equivalently convincing. For clarity, for the above 8 numbers, you only need 7 explanations.

Example explanation 1: $0.7 < \frac{5}{7}$ because $0.7 = \frac{7}{10} = \frac{49}{70} < \frac{50}{70} = \frac{5}{7}$

Example explanation 2: $\frac{1}{3} < 0.75$ because (*insert picture of pie with shading*)

2 Calculating tip and interest

(a) Suppose you receive a restaurant bill for \$75. Calculate the waiter's tip ...

- i. If you tip 15% (*hint:* 15% of \$75)
- ii. If you tip 20%
- iii. *Extra:* If you tip 18%

(b) Suppose Jose goes to a payday lender (sketchy loans) on January 1st to borrow \$100 with a 20% interest rate for each month late.

This means that, if Jose doesn't pay any of the \$100 dollars by February 1st, he now owes \$120 (*hint:* $1.2 * 100 = 120$) before March 1st. Further, if Jose doesn't pay any of the \$120 dollars by March 1st, he now owes \$144 (*hint:* $1.2 * 120 = 144$) before April 1st.

For the problem, suppose, instead, that Jose borrows \$200 on January 1st with a 50% interest rate for each month late. Suppose further that Jose gets laid off and makes no payments towards his \$200 loan.

- i. How much does Jose owe on April 1st?
- ii. *Extra:* If Jose pays \$100 on the 15th of each month, how many months does it take him to pay off his loan?

3 Rates and proportions

(a) If Jose folds 3 shirts in 5 minutes, how many complete shirts can Jose bake in 13 minutes?

- (b) If Jose eats 2 burritos per hour and Nishith eats 3 burritos per hour, then, as a team, how many burritos can Jose and Nishith eat per hour?
- (c) If Jose bakes 2 cakes per hour and Nishith bakes 3 cakes per hour and Barak bakes 5 cakes per hour, then, as a team, how many cakes can Jose and Nishith and Barak bake per hour? *Extra:* How many cakes can they bake in a bake off that lasts 3 hours? *Extra:* How long does it take them to bake 11 cakes?

4 Statistical thinking

- (a) Suppose we tag 100 buffalo and then release them back into the wild. If we fly a helicopter over their grasslands and 20 out of 100 buffalo we see are tagged, what would you estimate as the size of the buffalo population?
- (b) Suppose we tag 1000 buffalo and then release them back into the wild. Soon after, when we flew a helicopter over the grasslands, 100 out of 600 buffalo had tags. Further, suppose Jose shows up and each time he goes hunting, he kills 10 random buffalo. Assuming no population change, how many tagged buffalo get hunted after 7 hunting trips?

5 Cookie-cutter geometry

- (a) Calculate the area of a circle with diameter 2 cm. Recall that $Area = \pi * r^2$.
- (b) What is the area of a circle of radius 2 cm? Radius 4 cm? Radius 6 cm? In a sentence, explain how the area changes as we increase the radius?
- (c) Provide an explanation for why the formula of a trapezoid's area, $Area = \frac{1}{2}(B_1 + B_2) * H$, makes sense, where B_1 , B_2 , and H are the trapezoid's top base length, bottom base length, and height, respectively.
- (d) *Extra:* Suppose I have a loop of string with perimeter 20 cm. What are the dimensions of the rectangle with the largest area I can make with that string (and thumbtacks to hold the string in place)?

6 Geometry with a dash of rates and proportions

- (a) Suppose the target logo has three concentric circles, with diameters of length 2, 4, and 6 centimeters, respectively. What fraction of the area is red?
- (b) Suppose Jose is 6 tall and casts a 9 shadow. How long is the shadow of a 10 lamp post? What if he casts an 11 shadow?
- (c) At a restaurant a small burger costs \$9 and a large burger costs \$16. Assuming no discounts and equal heights of the circular burger patties, if the small patty has area 12π , what would you expect to be the area of the larger patty?

7 Geometry and statistical thinking

- (a) Consider a dartboard with two concentric circles of radius 5 cm and 10 cm. If I throw a dart at the dartboard, what is the probability that I will hit the inner circle?
- (b) If the dartboard has diameter 20 cm, how big should the inner circles diameter be if we want a 25% chance of hitting the inner circle? What about 50% chance?
- (c) Consider a sheet of paper with a circle of diameter 8 cm and a square of side length 8 cm. If 100 rain drops are falling, which shape would you estimate receives more rain? How many more drops?